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6. AUTHOR(S)

Hans-Georg Müller & Jane-Ling Wang

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Division of Statistics
University of California
Davis, CA 95616

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The research funded during this period led to more than 30 accepted or published papers in refereed statistical journals. Several graduate students were involved in the research during this period who contributed significantly to the success of the project as well as benefited form opportunities for educational and research achievements leading to the Ph.D degree (Sam Hawala, Kathryn Prewitt and Kai-Sheng Song). The research achievements for this project roughly fall into five areas of research, all of which are in nonparametric statistics. They cover a wide range of topics from the applied to the theoretical and have important implications for data analysis, as well as for the theory of Statistics.

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FINAL REPORT

Grant No. AFOSR-89-0386

Project Title: *Nonparametric Methods for Incomplete Data in Reliability and for Changepoints in Smooth Functions*

Period of Support: July 1, 1991 - June 30, 1994

PI/PD: Jane-Ling Wang

Co-Investigator: Hans-Georg Müller

1. SUMMARY OF COMPLETED PROJECT

The research funded during this period led to more than 30 accepted or published papers in refereed statistical journals. Several graduate students were involved in the research during this period who contributed significantly to the success of the project as well as benefited from opportunities for educational and research achievements leading to the Ph.D. degree (Sam Hawala, Kathryn Prewitt and Kai-Sheng Song). The research achievements for this project roughly fall into five areas of research, all of which are in nonparametric statistics. They cover a wide range of topics from the applied to the theoretical and have important implications for data analysis, as well as for the theory of Statistics. Note that some papers could be classified into several of these categories; the numbers in the following refer to the list of published papers funded by this grant, given in Section 5.1 below.

A. *Nonparametric inference for incomplete data subject to censoring/truncation.*

These research works have applications in theoretical and applied reliability. ([12], [14], [15], [16], [17], [25], [28]). A major theoretical result was for instance achieved in [15].

B. *Nonparametric estimation of density functions and failure rates for incomplete data.*

Important applications are in monitoring and quality control technology ([5], [11], [13], [24]). Particularly noteworthy is [24], producing an entirely data-based adaptive smoothing method for hazard and density functions and their derivatives under random censoring.

C. *Nonparametric analysis of discontinuities in hazard functions, regression curves and multidimensional surfaces.*

Applications of this research are in statistical image analysis and nonparametric change-point methods ([7], [26], [27], [29]). A particularly innovative method of change-point detection in the context of smooth regression functions was proposed in [7].

D. *Adaptive estimation of spectra and peaks in spectra for stationary processes, and of regression functions and surfaces.*

Applications are in signal processing and nonparametric curve estimation. ([1], [9], [19], [22]). A central result is [9].

E. *Topics in nonparametric regression, especially diagnostics for nonparametric regression and variance function modelling.*

Applications are in regression type problems ([2], [3], [4], [6], [8], [10], [18], [20], [21], [23], [30], [31]). An entirely new method of improved one- and multidimensional nonparametric regression is for instance [20].

2. COMPLETED RESEARCH

This section contains more detailed descriptions of the research achievements in the five main areas A-E of research which were funded by the grant.

2.1. Nonparametric inference for incomplete data subject to censoring/truncation (Area A). ([12], [14], [15], [16], [17], [25], [28]).

Weak and strong quantile representations for randomly truncated data with applications. Suppose that we observe bivariate data (X_i, Y_i) only when $Y_i \leq X_i$ (left truncation). Denote with F the marginal d.f. of the $X's$. In this paper we derive a Bahadur-type representation for the quantile function of the pertaining product-limit estimator of F. As an application we obtain confidence intervals and bands for quantiles of F.

On the Hajek projection for truncated and censored data. Large sample properties of the product-limit estimators for truncated or censored data are usually achieved via the empirical cumulative hazard function estimators. Hajek projection of the empirical cumulative hazard function estimator is derived for truncated data and expressed for censored data. It turns out that both projections are asymptotically $n^{1/2}$-equivalent but not equal to the respective influence curves. Weak convergencies of the empirical cumulative hazard processes are deduced accordingly.

A strong law under random censorship. Let $X_1, X_2, ...$ be a sequence of i.i.d. random variables with d.f. F. We observe $Z_i = \min(X_i, Y_i)$ and $\delta_i = 1_{\{X_i \leq Y_i\}}$, where $Y_1, Y_2, ...$ is a sequence of i.i.d. censoring random variables. Denote with $\hat{F}_n$ the Kaplan-Meier estimator of F. We show that for any F-integrable function φ, $\int \varphi d\hat{F}_n$ converges almost surely and in the mean. The result may be applied to yield consistency of many estimators under random censorship.

Strong representations of the survival function estimator for truncated and censored data with applications. A strong i.i.d. representation is obtained for the product-limit estimator of the survival function based on left truncated and right censored data.This extends the result of Chao and Lo (1988, *Ann. Statist.* **16**, 661-668) for truncated data. An improved rate of the approximation is also obtained on compact sets. Applications include density and hazard rate estimation. The advantage of the improved rate of the approximation is illustrated via kernel density estimation.

Multi-Sample U-Statistics for censored data. In this paper we prove almost sure convergence of multi-sample U-statistics under random censorship. As an application we obtain consistency of a new class of tests designed for testing about equality in distribution.

The jackknife estimate of a Kaplan-Meier integral. We derive an explicit formula for the jackknife estimate of a Kaplan-Meier integral. From this the asumptotic analysis of the jackknifed Kaplan-Meier process becomes straightforward. In a small simulation study it is demonstrated that jackknifing may lead to a considerable reduction of the bias.

M-estimators for censored data: Strong consistency. Let $\hat{F}_n(x)$ denote the Kaplan-Meier product-limit estimate for the life distribution function $F(x;\theta_0)$ based on randomly censored data. The M-estimator of θ_0 corresponding to a function ρ is defined to be the value of θ which minimizes $\int \rho(x;\theta) d\hat{F}_n(x)$. The strong consistency of M-estimators is studied. It is shown that most of the classical sufficient conditions based on ρ, such as Wald (1949), Kiefer and Wolfowitz (1956), Huber (1967) can be extended to randomly censored data. Two such extensions based on Perlman (1972) and Wang (1985) are illustrated in detail and applied to parametric, semi- and non-parametric classes.

2.2. Nonparametric estimation of density functions and failure rates for incomplete data (Area B). ([5], [11], [13], [24])

A comparison of adaptive hazard rate estimators for left truncated and right censored data. Left truncation and right censoring arise frequently in practice for life data. This paper is concerned with the estimation of the hazard rate function for such data. Two types of nonparametric estimators based on kernel smoothing methods are considered. The first one is obtained by convolving a kernel with a cumulative hazard estimator. The second one is in the form of a ratio of two statistics. Local properties including consistency, asymptotic normality and mean squared error expressions are presented for both estimators. These properties facilitate locally adaptive bandwidth choice. The two types of estimators are then compared based on their theoretical and empirical performances. The effect of overlooking the truncation factor is demonstrated through the Channing House data.

Nonparametric maximum likelihood estimation of an increasing hazard rate for uncertain cause-of-death data. In Kaplan-Meier estimation of the survival function for diseased animals the cause of death has be specified with certainty. When pathologists are unable to do so forced cause-of-death data can create substantial biases. For unidentifiable cause-of-death data we derive the nonparametric maximum likelihood estimator of the hazard rate due to the disease assuming it is increasing when the survival function without the disease is known or can be well-estimated. Strong consistency of the maximum likelihood estimator is also obtained. Such model arises in Carcinogenesis bioassy. For example, consider an experiment in which laboratory animals are followed and examined on death for the existence of a particular type of tumor and to determine if this was the cause of death. When diagnosis can be given with certainty the usual life-table or Kaplan-Meier (1958) approach is applicable and simplifies the statistical analysis. However, it often happened (Dinse (1986)) that the pathologists cannot assert the cause of death and forcing the cause of death to be specified may result in substantial bias (Racine-Poon and Hoel (1984)). Our approach does not require the knowledge of the cause of death and assumes instead that the survival function of death due to all other competing risks can be determined quite accurately from other sources. Such a model arises, for example, in engineering context when a new device is added to an existing system whose survival function is well understood and assumed to be known, and it is not possible or too costly to specify whether a failure of the new system is due to the new device or not. If the life distribution of the risk of interest has increasing hazard rate, the nonparametric maximum likelihood estimator of the hazard rates reduces to the solution of a nonstandard optimization problem. A solution is given in this paper in the form of the max-min formula together with a computation algorithm based on the maximum upper sets. Nonparametric maximum likelihood estimators for the distribution and density functions are then implied by the invariance principle. The proof of the strong consistency utilizes the total time on test transformation in a nonstandard way and generalized a useful lemma in isotonic procedures due to Marshall (1970).

Nonparametric estimation of hazard functions and their derivatives under truncation model. Nonparametric kernel estimators for hazard functions and their derivatives are considered under the random left truncation model. The estimator is of the form of sum of identically distributed but dependent random variables. Exact and asymptotic expressions for the biases and variances of the estimators are derived. Mean square consistency and local asymptotic normality of the estimators are established. Adaptive local bandwidths are obtained by estimating the optimal bandwidths consistently.

Hazard rate estimation under random censoring with varying kernels and bandwidths. We discuss the estimation of hazard rates under random censoring with the kernel method. Two practically relevant problems which occur when applying unmodified kernel estimators are boundary effects near the endpoints of the support of the hazard rate, and a substantial increase in the variance from left to right over the range of abscissae where the hazard rate is estimated. A new class of boundary kernels is proposed for the first problem. Explicit formulas for these kernels are developed, and it is shown that this boundary correction works well in practice. A data-adaptive varying bandwidth selection procedure is proposed for the second problem. ·This procedure generally will lead to increasing bandwidths near the left endpoint and towards the right endpoint and will lead to smaller integrated mean squared error of the hazard rate estimator as compared to a fixed bandwidth method. A practically feasible method incorporating the new boundary kernels and local bandwidth choices is implemented and illustrated with survival data from a leukemia study.

2.3. Nonparametric analysis of discontinuities in hazard functions, regression curves and multidimensional surfaces. (Area C). ([7], [26], [27], [29])

Change-points in nonparametric regression analysis. Estimators for location and size of a discontinuity or change-point in an otherwise smooth regression model are proposed. The assumptions needed are much weaker than those made in parametric models. The proposed estimators apply as well to the detection of discontinuities in derivatives and therefore to the detection of change-points of slope and of higher order curvature. The proposed estimators are based on a comparison of left and right one-sided kernel smoothers. Weak convergence of a stochastic process in local differences to a Gaussian process is established for properly scaled versions of estimators of the location of a change-point. The continuous mapping theorem can then be invoked to obtain asymptotic distributions and corresponding rates of convergence for change-point estimators. These rates are typically faster than $n^{-1/2}$. Rates of global L^p convergence of curve estimates with appropriate kernel modifications adapting to estimated change-points are derived as a consequence. It is shown that these rates of convergence are the same as if the location of the change-point was known. The methods are illustrated by means of the well-known data on the annual flow volume of the Nile River between 1871-1970.

Maximin estimation of multivariate boundaries. We consider the problem of estimating the location and size of a discontinuity in an otherwise smoooth multidimensional regression function. The boundary or location of the discontinuity is assumed to be a closed curve respective surface, and we aim to estimate this closed set. Our approach utilizes the uniform convergence of multivariate kernel estimators for directional limits. Differences of such limits converge to zero under smoothness assumptions, and to the jump size along the discontinuity. This leads to the proposal of a maximim estimator, which selects the boundary for which the minimal estimated directional difference among all points belonging to this boundary is maximized. It is shown that this estimated boundary is almost surely enclosed ina sequence of shrinking neighborhoods around the true boundary, and corresponding rates of convergence are obtained.

Change-point models for hazard functions. A review is presented of parametric and nonparametric models and corresponding estimation procedures for change-points in hazard functions where the data are possibly subject to random censoring. In particular, we discuss nonparametric models and the application of nonparametric smoothing techniques for change-point estimation and estimation of a hazard function when a change-point is present. Preliminary theoretical results are mentioned and a simulation study provides further insight.

Cube Splitting in Multidimensional Edge Estimation. Assume noisy measurements are available and that an edge or boundary is given which induces a partition of the domain into two subsets. The regression function on one subset is equal to a constant c_1, on the other subset to a constant c_2. Each measurement is made within a regular pixel. The problem we consider is the estimation of the edge or boundary curve (change curve), for the case that the domain is in $\Re^d$. We propose to seek boundary estimates as maximizers of a weighted squared difference statistic where we maximize over unions of cubes of aggregated pixels. Rates of almost sure convergence of this procedure are established. Its central advantage is its numerical feasibility, as the number of cubes of aggregated pixels to be investigated for inclusion in one of the partitioning sets can be kept small. A numerically efficient "cube splitting" ("CUSP") algorithm is suggested which implements this proposal: Start with an iteratively grown union of big cubes of aggregated pixels to find a first approximate edge/boundary estimate on a coarse level of approximation. Then split those cubes falling near the boundary into smaller cubes and check their allocation to one of the partitioning sets in order to obtain a more refined boundary estimate. This cube splitting (refinement) step may then be iterated until the desired level of resolution is achieved.

2.4. Adaptive estimation of spectra and peaks in spectra for stationary processes, and of regression functions and surfaces. (Area D). ([1], [9], [19], [22])

Applications of multiparameter weak convergence for adaptive nonparametric curve estimation. We give an overview on applications of weak convergence of stochastic processes to obtain adaptive nonparametric curve estimators through efficient data-based local bandwidth choices. We point out new developments based on multivariate time stochastic processes. Examples are multivariate curve estimates, where several local bandwidths are to be chosen for different coordinates, and estimates of local functionals of curves which can be expressed as maxima or zeros of local deviation processes and also depend on a bandwidth. As an illustration, we show that adaptive mode estimation for a probability density function is a consequence of weak convergence of a two-dimensional process in a bandwidth and a deviation coordinate. Various adaptive mode estimators are discussed.

Weak convergence and adaptive peak estimation for spectral densities. Adaptive nonparametric kernel estimators for the location of a peak of the spectral density of a stationary time series are proposed and investigated. They are based on direct smoothing of the periodogram where the amount of smoothing is determined automatically in an asymptotically optimal fashion. These adaptive estimators minimize the asymptotic mean squared error. Adaptivity is derived from the weak convergence of a two-parameter stochastic process in a deviation and a bandwidth coordinate to a Gaussian limit process. Efficient global and local bandwidth choices which lead to adaptive peak estimators and practical aspects are discussed.

Comment to "*Local regression: automatic kernel carpentry*". A discussion of a paper on weighted local least squares methods and comparisons with kernel methods.

Multiparameter bandwidth processes and adaptive surface smoothing. We derive a functional limit theorem for a sequence of bandwidth processes with multivariate time and show that the limit process is multivariate Gaussian. This theorem is then applied to show asymptotic efficiency of certain data-adaptive local bandwidth choices for kernel estimators of multivariate regression functions and their derivatives. The cases where optimal multivariate bandwidths exist as minimizers of leading mean squared error terms are characterized.

2.5. Methods for nonparametric regression, especially diagnostics for nonparametric regression and variance function modelling. (Area E). ([2], [3], [4], [6], [8], [10], [18], [20], [21], [23], [30], [31])

Discussion of "Transformations in density estimation". A discussion of how transformations in density estimation affect estimation near boundaries.

Smooth optimum kernel estimators near endpoints. Kernel estimators for smooth curves like density, spectral density or regression functions with known compact support require modifications when estimating near the endpoints of the support, both for practical and asymptotic reasons. The construction of such boundary kernels as solutions of a variational problem is addressed and expansions in terms of orthogonal polynomials are given, including explicit solutions for the most important cases. Based on explicit formulas for certain functionals of the kernels, it is shown that local bandwidth variation might be indicated near boundaries. Various bandwidth variation schemes and the impact of boundary modifications on cross-validation bandwidths are investigated in a Monte Carlo study.

Optimizing kernel methods: A unifying variational principle. We consider a variety of optimization problems connected with the choice of a kernel function. An example is the optimzation of kernels for estimating characteristic points of a curve which are the locations of extrema of higher order derivatives. We discuss the problems of finding 'optimal' kernels minimizing the asymptotic mean squared error in this context and that of 'minimum variance' kernels minimizing the asymptotic variance. The corresponding variational problems are analyzed by means of Jacobi representations and explicit solutions which are polynomials with compact support are obtained. It is then shown that in fact a variety of other variational problems connected with the choice of optimal kernel functions are equivalent to this problem. A general underlying variational principle is uncovered and investigated. The limiting case as the order of smoothness of the kernel tends to infinity is studies, leading to analytic kernel functions on $\Re$ for which an explicit Hermite representation is found. The kernels thus obtained provide a natural extension of the optimal polynomial kernels with compact support.

Goodness-of-fit diagnostics for regression models. For a fixed design regression model, we compare the fitting of parametric linear or nonlinear models by the least squares method with a model-free nonparametric approach via kernel estimates. One major problem for such a comparison is the necessary bandwidth choice for the nonparametric estimate, and a data-adaptive method for local bandwidth choice based on the parametric fit is proposed. As an application we consider comparison of both estimates and of corresponding estimates of derivatives at a finite number of preselected points. This leads to a test statistic which is asymptotically χ^2 distributed under the null hypothesis that the parametric model contains the underlying regression function g and has asymptotic power 1 under certain contiguous alternatives. As compared to other proposed goodness-of-fit procedures, this test does not depend on the subjective choice of a bandwidth. Practical issues and diagnostic plots are illustrated in a data application.

Ultraspherical polynomial, kernel and hybrid estimators for nonparametric regression. We discuss orthogonal polynomial series estimators for a regression function and its derivatives using weighted (and in particular Ultraspherical) polynomials as the orthonormal basis. We consider also a localized version of this method which is a hybrid between kernel and orthogonal polynomial approaches and can be shown to be a generalization of both. Our analysis follows from the observation that the orthogonal polynomial estimator has an equivalent kernel interpretation, where the bandwidth is fixed and the number of basis elements included in the orthogonal polynomial estimator corresponds to the order of the equivalent kernel function. This equivalence also yields a new construction for boundary kernels. The hybrid estimators are more flexible than both kernel and orthogonal polynomial estimators. In a study comparing finite integrated mean squared errors for various cases, the behaviour of the hybrid estimator compared to the orthogonal polynomial estimators were assessed and its superiority for some cases is indicated.

Preaveraged localized orthogonal polynomial estimators for surface smoothing and partial differentiation. We propose a multivariate smoothing method based on products of localized orthogonal polynomial series estimators for a smooth regression surface in the fixed design regression model. The estimation of partial derivatives is included. The proposed method provides for automatic and efficient boundary modifications near the edges of the surface, assuming that the boundary of the support of the regression function satisfies some regularity conditions. By allowing for a preaveraging step, the corresponding algorithms are speeded up considerably and are easy to implement. Computation of special boundary kernels, as required by the kernel method in order to avoid edge effects, is not necessary. It is shown that under sufficient smoothness assumptions, the global average mean squared error has the same optimal rate of convergence as the mean squared error at an interior point, i.e., that the boundary correction is asymptotically effective. The method depends on two smoothing parameters, one determining the amount of preaveraging, the other the amount of smoothing after preaveraging. Theoretical and practical bounds for the choice of these parameters are discussed. A Monte Carlo study based on a bivariate normal surface indicates that increasing the preaveraging parameter δ has a negative effect on the average mean squared error, which is not unexpected. On the other hand, larger values of δ are computationally more economical. The effects of boundary correction as compared to non-corrected estimates are investigated for the example of a quadratic surface. The numerical complexity of the proposed method is discussed. The methods are demonstrated and compared to kriging for two data sets, one on non-uniformly measured ground-water levels in Arizona, the over on cover-clay thickness data from Iran measured on a regular mesh. The two data analyses include regular and irregular designs and supports and seem to indicate that the method works well in particular when compared to kriging.

On variance function estimation with quadratic forms. We propose a class of general quadratic forms in the dependent variable in order to estimate the variance function in a non-parametric heteroscedastic fixed design regression model. It is shown that these estimators achieve improved rates of convergence for the mean squared error as compared to estimators that were considered before. Besides results on consistency and rates of convergence, also the leading terms of the asymptotic mean squared error are obtained for some important cases. Several interesting special estimators are discussed in more detail.

Identity reproducing multivariate nonparametric regression. Nonparametric kernel regression estimators of the Nadaraya-Watson type are known to have an undesirable bias behavior. We propose a general technique to improve the bias of any given multivariate non-parametric regression estimator based on the requirement that the identity function should be reproduced, which is achieved by means of an identity reproducing transformation of the predictor variable. The asymptotic distribution of the identity reproducing version of the Nadaraya-Watson estimator is derived and is compared with that of the untransformed Nadaraya-Watson estimator. It is demonstrated by means of a Monte Carlo study that the asymptotic improvements are noticeable already for small sample sizes.

On boundary kernel method for nonparametric curve estimation near endpoints. Kernel estimators for nonparametric function estimation are affected by boundary effects when estimating near an endpoint of the support of the function. A general construction for boundary kernels is presented, which allows to remove these edge effects. It is shown that common kernel functions which satisfy some mild requirements can be derived as the solution of a variational problem involving a certain weight function. For the solutions of this variational problem, an explicit representation in polynomials which are orthogonal with respect to this "associated weight function" is found; thus any common kernel function can be represented as a product of an "associated weight function" and an orthogonal expansion. It is demonstrated how this variational problem and its solution can be extended to cover boundary kernels. The resulting explicit construction of boundary kernels includes kernels with compact as well as noncompact support, and examples are presented demonstrating the corresponding boundary kernels for compactly supported polynomial kernels, Gaussian kernels with unbounded support, and related analytical kernel functions.

Asymptotics for nonparametric regression. This paper reviews and compares some of the basic ideas in nonparametric regression with Mahalanobis' Fractile Graphical Analysis. New results concern a new proof of known properties of the "error areas" in Fractile Graphical Analysis using concomitants and a discussion of the properties of fractile graphs as a method of nonparametric regression (Fractile Regression). Furthermore, a general result for the local asymptotic distribution of real-valued functions with arguments which are functionals formed by weighted averages of the data is provided. This result is applied to derive local distributions for various nonparametric regression type estimates, including estimators for derivatives.

Orthogonal polynomial and hybrid estimators for nonparametric regression. This algorithm calculates a nonparametric regression curve and derivative estimate based on a hybrid formulation between kernel and orthogonal polynomial methods (Azari, Mack, Müller, 1992) The estimators depend on two smoothing parameters which have to be provided by the user and contain both kernel and orthogonal polynomial estimators as special cases. as well as an asymptotic equivalent of polynomial regression. Automatic boundary corrections are included.

Estimating direction fields in autonomous equation models, with an application to system identification from cross-sectional data. We consider a situation where 'cross-sectional' observations follow an underlying 'longitudinal' model with a population mean function. Measurements of the mean function values without noise and of derivatives with noise are available, which are made at unknown "observation times". The population mean function lies in a nonparametric smoothness class, and while not fully identifiable, is a trajectory in the direction field of an autonomous ordinary differential equation. An efficient reconstruction of that field is proposed which proceeds from nonparametric estimation of the function driving the differential equation. Rates of convergence are investigated. A simulation is included showing the recovery of the mean function. Signal and noise are chosen to be typical for T4-cell counts in a small cohort of undated HIV-seroconverters, and the reconstruction of the mean function is seen to be satisfactory.

3. Ph.D. Students

Ph.D. Thesis completed.

1. Kathryn Prewitt (1992). Now at Arizona State University.
2. Kai-Sheng Song (1993). Now at Texas A&M University.
3. Sam Hawala (1994). Now at University of St. Thomas, St., Paul, Minnesota.

Continuing Ph.D. Theses on Related Topics with likely year of completion.

1. Jerome Braun (1996)
2. William Capra (1995).

4. INVITED TALKS

Invited Talks at Conferences/Research Institutes.

Talks at scheduled Colloquia at Universities are not listed.

Hans-Georg Müller:

Workshop on Trends in Curve Estimation, University of Heidelberg, Germany, 1991.
IMS-ASA Regional Meeting, Houston, TX, 1991.
IMS-WNAR Regional Meeting, Corvallis, OR, 1992.
AMS-IMS-SIAM Summer Research Conference on Change-point problems, Mt. Holyoke, MA, 1992.
Workshop on Change-point Analysis and Empirical Reliability, Carleton University, Ottawa, 1992.
Meeting on Mathematical Statistics, Mathematical Research Institute, Oberwolfach, Germany, 1993.
International Summer School on Function Estimation, Santiago de Compostela, Spain, 1993.
Statistics Days, Marburg, Germany, 1993.
Workshop on New Aspects of Numerical Analysis, Stresa, Italy, 1993.
International Chinese Statistical Association Conference, Taipei, Taiwan, 1993.
International Meeting on Lifetime Data Models in Reliability and Survival Analysis, Harvard University, MA, 1994.
Interface Meeting Statistics and Computing, Research Triangle Park, NC, 1994.

Jane-Ling Wang:

Mathematical Sciences Research Institute at Berkeley, CA,1992.
AMS-IMS-SIAM Joint Summer Research Conference on Change-point Problems, South Hadley, MA, 1992.
Workshop on Change-point Analysis and Empirical Reliability, Carleton University, Ottawa, 1992.
International Seminar on "Functional Estimation", Santiago de Compostela, Spain, 1993.
International Chinese Statistical Association Conference, Taipei, Taiwan, 1993.
International Meeting on Lifetime Data Models in Reliability and Survival Analysis, Harvard University, MA, 1994.
NSF Co-sponsored International Workshop on "New Directions in Curve Estimations", 1994.

5. PUBLICATIONS

5.1. Publications during Funding Period.

This list contains all accepted or published papers, which were influenced by research efforts, which were fully or partly funded by the AFOSR grant during the review period. They are listed chronologically. Several preprints documenting further research were produced as well, but are not listed here.

[1] H.G. Müller, K. Prewitt (1991), Applications of multiparameter weak convergence for adaptive nonparametric curve estimation. *Nonparametric Functional Estimation and Related Topics.* Proceedings of NATO Advanced Study Institute, Spetses, 1990, Ed. G.G. Roussas, Kluwer Academic Publishers, 141-166.

[2] H.G. Müller, H. Zhou (1991), Discussion of "Transformations in density estimation" by M. Wand, J.S. Marron and D. Ruppert. *J. American Statistical Association* **86**, 356-358.

[3] H.G. Müller (1991), Smooth optimum kernel estimators near endpoints. *Biometrika* **78**, 521-530.

[4] B. Granovsky, H.G. Müller (1991), Optimizing kernel methods: A unifying variational principle. *International Statistical Review (Part B)* **59**, 373-388.

[5] U. Uzunogullari, J.L. Wang (1992), A comparison of adaptive hazard rate estimators for left truncated and right censored data. *Biometrika* **79**, 297-310.

[6] H.G. Müller (1992), Goodness-of-fit diagnostics for regression models. *Scandinavian J. Statistics* **19**, 157-172.

[7] H.G. Müller (1992), Change-points in nonparametric regression analysis. *Annals of Statistics* **20**, 737-761.

[8] A.S. Azari, Y.P. Mack, H.G. Müller (1992), Ultraspherical polynomial, kernel and hybrid estimators for nonparametric regression. *Sankhyā A* **54**, 80-96.

[9] H.G. Müller, K. Prewitt (1992), Weak convergence and adaptive peak estimation for spectral densities. *Annals of Statistics* **20**, 1329-1349.

[10] A.S. Azari, H.G. Müller (1992), Preaveraged localized orthogonal polynomial estimators for surface smoothing and partial differentiation. *J. American Statistical Association* **87**, 1005-1017.

[11] H. Mukerjee, J.L. Wang (1993), Nonparametric maximum likelihood estimation of an increasing hazard rate for uncertain cause-of-death data. *Scandinavian J. Statistics* **20**, 17-34.

[12] U. Güler, W. Stute, J.L. Wang (1993), Weak and strong quantile representations for randomly truncated data with applications. *Statistics & Probability Letters* **17**, 139-148.

[13] U. Güler, J.W. Wang (1993), Nonparametric estimation of hazard functions and their derivatives under truncation models. *Annals of the Institute of Statistical Mathematics* **45**, 249-264.

[14] U. Güler, J.W. Wang (1993), On the Hajek projection for truncated and censored data. *Sankhya A*, **55**, 66-79.

[15] W. Stute, J.L. Wang (1993), A strong law under random censorship. *Annals of Statistics* **21**, 1591-1607.

[16] I. Gijbels, J.L. Wang (1993). Strong representations of the survival function estimator for truncated and censored data with applications. *J. Multivariate Analysis* **47**. 210-229.

[17] W. Stute, J.L. Wang (1993), Multi-Sample U-Statistics for censored data. *Scandinavian J. Statistics* **20**, 369-374.

[18] H.G. Müller, U. Stadtmüller (1993), On variance function estimation with quadratic forms. *J. Statistical Planning Inference* **35**, 213-231.

[19] H.G. Müller (1993), Comment to "Local regression: automatic kernel carpentry" by Hastie and Loader *Statistical Science* **8**, 134-139.

[20] H.G. Müller, K.S. Song (1993), Identity reproducing multivariate nonparametric regression. *J. Multivariate Analysis* **46**, 237-253.

[21] H.G. Müller (1993), On the boundary kernel method for nonparametric curve estimation near endpoints. *Scandinavian J. Statistics* **20**, 313-328.

[22] H.G. Müller, K. Prewitt (1993), Multiparameter bandwidth processes and adaptive surface smoothing. *J. Multivariate Analysis* **47**, 1-21.

[23] P.K. Bhattacharya, H.G. Müller (1993), Asymptotics for nonparametric regression. *Sankhyā A* **55**, 420-441.

[24] H.G. Müller, J.L. Wang (1994), Hazard rate estimation under random censoring with varying kernels and bandwidths. *Biometrics* **50**, 61-76.

[25] W. Stute, J.L. Wang (1994), The jackknife estimate of a Kaplan-Meier integral. *Biometrika* **81**, 602-606.

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